UNSTEADY-STATE THERMOPHYSICAL MODELS FOR DETECTION OF PHYSIOLOGICAL AND PATHOLOGICAL CONDITIONS OF THE KIDNEY BY METHODS OF ACTIVE DYNAMIC IR THERMOGRAPHY

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Unsteady-state processes of biological heat transfer are simulated and their numerical analysis is made to find out whether it is possible to determine thermal properties of the kidney by methods of active dynamic IR thermography. Criteria are developed for interpretation of dynamic IR thermal charts for medical diagnosis that noticeably increase its accuracy and reliability in comparison with the present potentialities of static IR thermography. The analysis performed allows thermal diagnostic interpretation of the dynamics of the functioning of the kidney in normalcy and in a particular pathology.

Introduction. A living organism is an open energy system that is in thermodynamic equilibrium with the environment. The space-time distribution of natural physical fields and radiation of the human organism is a noninvasive source of information about its condition [1].

Nevertheless, in recent years various visualization methods based on external actions on the biological object have been increasingly widely used for recording and measurement of newly appearing physical fields of the object. However, information about the distribution of these fields often turns out to be different from their actual quasisteady distribution [1] at still unknown safe doses of these actions for the organism.

Meanwhile, the potentialities of noninvasive methods of diagnostic visualization of natural radiation of biological objects are far from being exhausted [1, 2]. Among them, we should mention foremost computer IR thermographic methods that provide medical diagnostic data that are unique in their informativeness [1, 3, 4]. However, in recent years clinical interest in these methods seems to be declining [1]. A possible source of this decline is the complexity and, frequently, ambiguity of the interpretation of the human temperature field [1, 5], which results in the impossibility of clear-cut differentiation of radiation emitted from the skin surface and from particular organs that are to be diagnosed. Moreover, there are no adequately developed expert systems for computer IR thermography, and the likelihood of making an inadequate diagnosis is high due to the high subjectivity of the operator physician's assessment of IR thermograms.

Results obtained in [6] expand the diagnostic capabilities of IR thermograms as indicators of the distribution of a variable blood flow in concrete clinical situations. In spite of the analysis of thermal charts of the kidney suggested in that work that increases the potentialities of conventional static IR thermography, ambiguity of interpretation of thermal charts is still possible and obviously not infrequent. Moreover, clinical diagnostic methods of static IR thermography are far from always being effective.

Methods of active dynamic thermography are much more informative in comparison to static thermography [4, 7]. The former combine procedures used to investigate not only spatial but also spatial-temporal changes in IR radiation from the skin surface caused by physicochemical actions that are usually applied to increase contrast and to obtain a more detailed picture of multifunctional structures in thermal charts. The possibility of obtaining much more information and increasing the reliability of thermographic examinations in comparison with static thermography is the main advantage of methods of active dynamic IR thermography [2, 4, 7]. However, these

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Fig. 1. Kinetics of the development of quasisteady states of the kidney arising as a result of instantaneous disturbance of the initial blood flow $w^{st} = 0.4 \text{ ml/(g·min)}$ at $h = 273 \text{ W/(m}^{2.0}\text{C})$ and different values of w (ml/(g·min)): 1) 0.32, 2) 0.48, 3) 0.51. T_s , ^oC; t, min.

methods are still rarely used and are not adequately assessed [7]. Their mathematical basis is not reported in the literature.

Mathematical Formulation of the Problem. In view of the situation put together, in the present work we consider a model of the thermal conditions of a particular organ, the kidney, subjected to an external action. The formulation of the problem proceeds from the need to determine the role of various factors and their influence on the blood flow and heat generation in the kidney and on the potentialities of IR visualization. Because of this, we restrict ourselves to the use of a one-dimensional geometric model of the kidney [3, 6]. From the four main groups of approaches in theoretical analysis with account for the blood flow in renal tissue [2], we have chosen one that assumes that the blood-flow velocity remains unchanged under physicochemical actions applied to the kidney and depends only on the type of biological tissue and its physiological condition.

In constructing a thermophysical model of the kidney, we use Penn's classical unsteady-state biothermal equation $\{2, 8\}$ in dimensionless variables, assuming that the thermophysical parameters do not depend on the temperature or time in the ranges considered. Heat transfer in the kidney is described by means of boundary-value problem (1)-(3) formulated in [6]. Here it is assumed that at the initial time the spatial distribution of temperature in the renal tissues is known:

$$\Theta(\xi,\tau)\big|_{\tau=0} = \Theta^{s_1}(\xi), \ \xi \in [0;1],$$
(1)

where $\Theta^{st}(\xi)$ is the solution of the steady-state boundary-value problem with the parameters $\beta^{st}(\xi)$ and $Q^{st}(\xi)$ that is consistent with the unsteady-state problem considered.

Results and Discussion. Solution of the present mixed problem by the Fourier method [9] resulted in the following expression for the temperature distribution in renal tissues:

$$T(x, t) = T_{a} - (T_{a} - T_{\infty}) \left[\sum_{n \ge 1} (A_{n} (\beta^{2}) K^{\text{st}} K_{n} + F_{n} (\beta^{2}, Q) D_{n}) \times (1 - \exp(\alpha_{n} t)) \cos y_{n} \frac{x}{L} + K^{\text{st}} \cosh \beta^{\text{st}} \frac{x}{L} - Q^{\text{st}} \right].$$
(2)

Already at t > 1 sec heat transfer in the kidney is described quite accurately by an approximate solution of the formulated problem that includes only the first term from the infinite sum entering into expression (2). In numerical calculations we took into consideration the resolution of modern computer IR systems (0.03°C) [1] and the possible frequency of measurements (at least every minute). The thermal and physiological characteristics of the kidney used in the calculations are presented in [6, 8].



Fig. 2. Surface temperature T_s after instantaneous disturbance of the initial blood flow $w^{st} = 0.6 \text{ ml/(g·min)}$ at $h = 14 \text{ W/(m}^{2.0}\text{C})$ and different values of w (ml/(g·min)): 1) 0.78, 2) 0.72, 3) 0.66.

It is known that in the case of any pathology of the kidney, regulatory processes that affect the behavior of the renal blood flow appear disturbed. The disturbance of the blood flow changes the heat generation of the organ and the local heat balance, which is reflected in the IR thermal chart [4, 10].

Figure 1 shows curves that indicate that disturbances of the blood flow that are rather close in numerical value result in new heat conditions appearing in the kidney whose reliable diagnostic characteristic can be, for example, the time of their development t_{qst}^0 . In the cases considered here the time t_{qst}^0 varies in the range of 5 to 9 min. However, when the blood flow increases or decreases by the same value Δw , the time index t_{qst}^0 takes very close values, which is a result of the linear formulation in time for the biological heat transfer problem. Therefore, for characterization of newly developing spatial-temporal distributions of temperature specified by the thermal conditions of the kidney, use of the temperature index $T_s(t_{qst}^0)$ as an additional diagnostic criterion obviously decreases the ambiguity of interpretation of IR thermographic data. Thus, the physiological problem of creating disturbances of the blood flow in concrete biological tissues that should be controlled and limited in the time of their action arises.

For this purpose, in active IR thermography, loading medication tests are used. Their administration is accompanied by changes in glomerular filtration and blood flow. In particular, intravenous injection of diuretics results in a 30% increase in the renal blood flow [11, 12]. As a result, development of new quasisteady thermal conditions of the kidney is observed. Here the obtained time and corresponding temperature readings are quite reliable, and they differ in informativeness from the values of T_s recorded by methods of static IR thermography.

It is evident that, having appeared, the new quasi-equilibrium state of the kidney is retained only for some time interval t_{qst} because the thermal conditions of the organ actually return to the initial ones [4]. The behavior of the temporal distribution $T_s(t)$ described above is shown in Fig. 2, which is plotted on the basis of calculations carried out following the suggested model. Here the times that characterize relaxation processes involved in active IR thermography depend on the conditions of application of the action to the kidney and are determined experimentally. The present results determine not only their qualitative but also their quantitative relation to the biomechanical and physiological conditions of the organ in specific clinical situations and to the conditions of the action applied to the kidney to change its heat conditions.

Thus, the present simulation shows that an important advantage of medical diagnosis made by methods of active dynamic IR thermography is the possibility of considering at least four additional, basically new parameters that objectively reflect the physiological conditions of an organ, in particular, the kidney, in normalcy and in pathology and that are not used in static IR thermography. These parameters are as follows (Figs. 1, 2): the time t_{qst}^0 of development of a new quasi-equilibrium thermal state of the organ subjected to the action, the duration t_{qst} of this state, the temperature $T_s(t_{qst}^0)$, and the time of relaxation t_{rel} to the initial thermal state. It is evident that



Fig. 3. Kinetics of the development of quasisteady states for a two-layer model of the kidney at h = 14 W/(m^{2.o}C), a constant blood flow $w_2^{st} = w_2 = 0.32$ ml/(g·min) in the outer layer, and an initial value $w_1^{st} = 2$ ml/(g·min) of the blood flow in the inner layer of the kidney under instantaneous disturbances of w_1^{st} to values of w_1 (ml/(g·min)): 1) 0.4, 2) 0.8, 3) 1.2, 4) 1.6, 5) 2.5.

analysis of dynamic IR thermal charts can be performed in terms of other criteria that are calculated from the suggested thermal models or modifications of them. It should be noted that in any case, quantitative and even qualitative criteria for dynamic IR thermography are more informative than the one parameter that is ordinarily used in static IR diagnosis.

The diagnostic potentialities of IR thermography that were found in the analysis of the thermal models can be expanded in view of the fact that the structure of organs or tissues in a living organism that are to be investigated consist of many layers. The inhomogeneity of their properties follows from thermal properties that are different in different cross sections of the blood flow etc. The kidney is really a multilayer organ whose tissues have different morphofunctional characteristics [4]. Therefore, it is of interest to analyze nonequilibrium heat-transfer processes in multilayer models of the kidney. It is considered as a two-layer organ that has an inner layer that is formed by renal parenchyma and a surface layer. In this case, the thermophysical model can be reduced to the following mixed boundary-value problem:

$$\frac{\partial \Theta_i}{\partial \tau} = \frac{\partial^2 \Theta_i}{\partial \xi^2} - \beta_i^2 \Theta_i - \beta_i^2 Q_i, \quad i = \overline{1, 2}; \quad \xi \in [\xi_i, \xi_{i+1}], \quad (3)$$

$$\Theta_{i}(\xi,\tau)|_{\tau=0} = \Theta_{i}^{\text{st}}(\xi), \ \xi \in [\xi_{i},\xi_{i+1}],$$
(4)

$$\frac{\partial \Theta_1}{\partial \xi} \bigg|_{\xi=0} = 0, \quad k_2 \frac{\partial \Theta_2}{\partial \xi} \bigg|_{\xi=1} = \operatorname{Bi} \left(1 - \Theta_2(\xi, \tau) \right|_{\xi=1},$$

$$\Theta_1(\xi, \tau) \bigg|_{\xi=\xi_2} = \Theta_2(\xi, \tau) \bigg|_{\xi=\xi_2}, \quad k_1 \frac{\partial \Theta_1}{\partial \xi} \bigg|_{\xi=\xi_2} = \left|k_2 \frac{\partial \Theta_2}{\partial \xi} \right|_{\xi=\xi_2}.$$
(5)

A solution of problem (3)-(5) can be found in a general form with the use of the Laplace operational method [13]. Direct application of the Laplace transform to system (3) with initial (4) and boundary (5) conditions gives a system of inhomogeneous linear ordinary differential equations with inhomogeneous external boundary conditions and splice boundary conditions. This boundary-value problem admits writing the solution in an explicit form. Application of expansion formulas [9] gives a general formula of the solution of the present problem in the form of infinite sums of series. These expressions are too bulky to be presented here. To obtain final numerical and

analytical solutions to problem (3)-(5), it is necessary to find roots of the transcendental characteristic equations [13] for the corresponding set of thermal parameters. In numerical calculations, the infinite sums in the solution are approximated by finite ones, and with account for the necessary accuracy of the solution at t > 1 sec, it is sufficient to take only the first terms in the series [13]. Some results of these numerical solutions are shown in Fig. 3. In the cases considered the blood flow of the renal parenchyma underwent an instantaneous disturbance caused by an external action applied to the kidney. It follows from the obtained solution of problem (3)-(5) and the results given in [6] that the temperature indices are very sensitive even to rather small (0.1-0.2 ml/(g·min)) changes in the blood flow rate in the surface layer of the kidney. A comparison of the results obtained in [6] and the data shown in Fig. 3 shows that the behavior of the blood flow and, consequently, the thermal conditions in the renal parenchyma are satisfactorily and more accurately diagnosed from thermographic indices in dynamic IR thermography than in static IR thermography.

Conclusions. Methods of active dynamic IR thermography provide more extensive and reliable information about physiological conditions of the kidney than methods of static IR thermography and increase the potentialities of diagnostic visualization of the pathogenesis of renal diseases since many of them are accompanied by disturbances of the blood flow and the mechanism of thermal regulation. Therefore, as is shown here on thermal models, thermal visualization of these disturbances can be carried out by methods of active IR thermography and can be a basis for creation of medical databases in differential diagnosis of renal pathologies.

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NOTATION

T, T_a , T_∞ , temperature of tissues, arterial blood, and the environment, respectively; *c*, *c*_b, specific heat of renal tissues and blood, respectively; *k*, specific thermal conductivity of tissue; ρ , ρ_b , density of tissues and blood, respectively; *w*, perfusion rate of blood; Q_0 , power of the heat-generation source in tissue that provides its temperature conditions in normalcy pathology, or forced external heating in the case of hyperthermia or controlled heat therapy; Bi = hL/k, Biot number; T_s , surface temperature; *a*, *b*, thickness of the parenchyma and surface layer of the kidney, respectively; $\{y_n\}_{n \ge 1}$, roots of the equation *y* tan *y* = Bi; $\Theta = \Theta(\xi, \tau) = (T_a - T)/(T_a - T_\infty)$; $\xi = x/L$; L = a + b; $\tau = kt/L^2\rho c$; $\beta^2 = \beta^2(\xi) = L^2 c_b \rho_b \rho w(\xi)/k$; $Q = Q(\xi, \tau) = (Q_0(\xi, \tau)\rho)/(\beta^2 k(T_a - T_\infty))$; $\xi_1 = 0$, $\xi_2 = a/L$, $\xi_3 = 1$; $\Theta^{st}(\xi) = 4Bi/(\beta^{st} \sinh \beta^{st} + Bi \cosh \beta^{st})$ (ch $\beta^{st}\xi$); $\Theta_i^{st}(\xi) = 4Bi/(\beta_i^{st} \sinh \beta_i^{st} + Bi \cosh \beta^{st})$, $\xi \in [\xi_i, \xi_{i+1}], i = \overline{1,2}$; $||X||_n^2 = (y_n^2 + Bi^2 + Bi)/2(y_n^2 + Bi^2)$; $\alpha_n = (k/L^2\rho c)(y_n^2 + \beta^2)$; $K^{st} = Bi(1 + Q^{st})/(\beta^{st} \sinh \beta^{st} + Bi \cosh \beta^{st})$; $D_n = \sin y_n^2/y_n^2 ||X||_n^2$; $A_n(\beta^2) = [(\beta^{st})^2 - \beta^2]/(y_n^2 + \beta^2)$; $\Gamma_n(\beta^2, Q) = \beta^2(Q^{st} - Q)/(y_n^2 + \beta^2)$; $K_n = (y_n \cosh^{st} \sin y_n + \beta^{st} \sinh \beta^{st} \cos y_n)/((y_n^2 + (\beta^{st})^2) ||X||_n^2)$.

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